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Multiple Metamers: Preserving Color Matches under Diverse Illuminants

*This article extends matrix-**R** operations, developed by Cohen and Kappauf in 1982 and 1985 and by Cohen in 1988, to create generalized metameric object color stimuli, here termed multiple metamers, that remain metameric with respect to several different illuminants. As discussed in prior literature, matrix-**R** operations resolve any radiometric function (specifying a color stimulus) into a fundamental color stimulus and a residual or metameric black, as hypothesized by Gunter Wyszecki in 1953. The fundamental is sensed by the visual system while the residual is ignored. Metameric color stimuli comprise the same fundamental but different residuals. Thus, matrix-**R** operations easily construct metameric suites under an equal-energy source or under any other illuminant whose physical specification is incorporated into the radiometric function of the color stimulus entering the visual system. In the present article, matrix-**R** operations are extended to create metameric stimuli that theoretically remain metameric with respect to an arbitrary number of sequential illuminants. While the stimuli remain metamers under different illuminants, the color sensation evoked with respect to one illuminant is generally different from the color sensation evoked with respect to another illuminant. Examples demonstrate multiple metamers with respect to CIE standard illuminants A and C and with respect to four artificial illuminants. In principle, extended matrix-**R** operations create multiple metamers with respect to different observers, or with respect to any combination of observers and illuminants.*

Introduction

This article further explores matrix-**R** operations, as developed by Cohen and Kappauf[1–2] and Cohen.[3] These prior articles reinterpret color and color mixture in terms of Euclidean geometry and the projection operator of linear algebra.[4–6] Portions of these prior articles appertain to the generation of metamers using the Wyszecki hypothesis. The present article extends matrix-**R** operations to generate suites of "multiple metamers" that remain metamers with respect to different illuminants. In the following discussion, a color stimulus is specified by a radiometric function; we shall sometimes refer to the radiometric function as the color stimulus itself.

The Wyszecki hypothesis, proposed by Gunter Wyszecki in 1953, states that any color stimulus comprises two parts, a fundamental color stimulus (or fundamental) and a residual (or metameric black). The fundamental carries all information for the color sensation and is transmitted by the color processing mechanism. The residual is undetected and is ignored by the color processing system. Members of a metameric suite comprise the same fundamental but different residuals, so that the addition of a residual to any member of a metameric suite creates still another member.

Suites of metameric color stimuli are generated easily by isolating the fundamental of a color stimulus and adding residuals from other color stimuli. Note that the fundamental, although itself a color stimulus, is a very special member (indeed, the host) of the metameric suite. Matrix-**R** operations are a conceptually compelling technique for the generation of metamers, in contrast to many other techniques described in prior literature.[7–12]

Let $N \times 3$ matrix **A** be any triplet of color-matching functions, as $\bar{x},\bar{y},\bar{z}$, or $\bar{b},\bar{g},\bar{r}$. The number of rows, N, is the arbitrary number of passbands chosen to represent the visual spectrum. Then, $N \times N$ matrix **R** is defined as $\mathbf{R} = \mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'$, where prime denotes transpose. Matrix **R** is invariant, that is to say, the same matrix **R** emerges from any linear transform of the **A** matrix. For any $N \times 1$ column matrix (vector) representing the radiometric function of color stimulus **N**, the corresponding fundamental, $\mathbf{N}^*$, is computed as $\mathbf{N}^* = \mathbf{RN}$. **B** is the residual of color stimulus **N**, and $\mathbf{N} - \mathbf{N}^* = \mathbf{B}$. The tristimulus values of **N** and $\mathbf{N}^*$ are identical and the tristimulus values of **B** are always 0,0,0.

Radiometric functions are N-dimensional; their specification requires N numbers. Fundamentals are special radiometric functions that require only three numbers for unique specification, while residuals require N–3 numbers. In stricter mathematical terms, we may say that an arbitrary N-dimensional radiometric function may be resolved into two components residing in separate subspaces that are mutual orthogonal complements. Although both fundamentals and residuals are radiometric functions comprising N values, their respective subspaces are very different; fundamentals lie in three-dimensional fundamental color space, and residuals lie in (N–3)-dimensional residual space.

Matrix-R Operations with Respect to a Single Illuminant

Matrix-**R** operations model the human visual color processing mechanism and are unrelated to the origin of the color stimulus, in particular, the nature of the illuminant or the reflective properties of the colored object. To generate multiple metamers, we modify the projection matrix **R** to incorporate the effect of a given illuminant.

For convenience, we define the characteristic, unbiased color stimulus, **N**, as the radiometric function emitted from an object under an equal energy illuminant. In contrast, biased color stimuli are radiometric functions emitted from the same object under any other illuminant, **W**. The usual representation of an illuminant is a series of N numbers. For our purposes, the illuminant is presented as a diagonal weighting matrix **W** that operates on the unbiased color stimulus, **N**, to give the biased color stimulus, **WN**.

The triplet of tristimulus values, **T**, of a biased stimulus is computed in the usual fashion: $\mathbf{T} = \mathbf{A}'(\mathbf{WN})$. It is sometimes useful to absorb the illuminant into the color matching functions by shifting the parentheses: $\mathbf{T} = (\mathbf{A}'\mathbf{W})\mathbf{N}$. Matrices (**WA**), which appear in the literature as mathematical conveniences,[13–14] are unrelated to the color processing mechanism because, in fact, the effect of the illuminant occurs *before* the stimulus enters the visual system. The columns of matrix **A** are conventional, unbiased color-matching functions; hence, the columns of matrix (**WA**) may be called biased color-matching functions. From purely physical considerations, it should be noted that the illuminant carries the power (irradiance) and the colored object reflectance modifies the energy levels at each wavelength. In associating **W** with the **A** matrix, we have split **W** into two parts: the scalar portion is absorbed into the **A** matrix; the units are absorbed into the object reflectance to give the unbiased color stimulus, **N**.

We seek a pair of unbiased color stimuli, $\mathbf{N}_1$ and $\mathbf{N}_2$, that are metamers after bias by a specific illuminant, say $\mathbf{W}_\alpha$. Since metamers have identical tristimulus values, by definition $\mathbf{A}'(\mathbf{W}_\alpha\mathbf{N}_1) = \mathbf{A}'(\mathbf{W}_\alpha\mathbf{N}_2)$. This equation may be rewritten as $\mathbf{A}_\alpha'(\mathbf{N}_1 - \mathbf{N}_2) = 0$, where $\mathbf{A}_\alpha = \mathbf{W}_\alpha\mathbf{A}$ is a triplet of biased color matching functions.

Projection matrix theory facilitates solution of the equation $\mathbf{A}_\alpha'(\mathbf{N}_1 - \mathbf{N}_2) = 0$ or $\mathbf{A}_\alpha'\mathbf{\Delta N} = 0$. Accordingly, we form projection matrix $\mathbf{P}_\alpha$ as $\mathbf{I} - \mathbf{A}_\alpha(\mathbf{A}_\alpha'\mathbf{A}_\alpha)^{-1}\mathbf{A}_\alpha'$, where **I** is the identity matrix. Any linear combination of the columns (or rows) of $\mathbf{P}_\alpha$ is the desired solution of $\mathbf{A}_\alpha'\mathbf{\Delta N} = 0$. Linear combinations of matrix rows or columns are expressed by matrix multiplication. Consequently, the general solution is $\mathbf{\Delta N} = \mathbf{P}_\alpha\mathbf{N}$, where **N** is an arbitrary set of N numbers.

A general statement can be made: Any pair of unbiased color stimuli that differ by $\mathbf{P}_\alpha\mathbf{N}$, where **N** is arbitrary, becomes metameric after bias by $\mathbf{W}_\alpha$. To obtain a *specific* solution pair, we add an arbitrary radiometric function, $\mathbf{N}_a$, to $\mathbf{P}_\alpha\mathbf{N}_b$, where $\mathbf{N}_b$ is an arbitrary set of N numbers. By selecting many different $\mathbf{N}_b$ vectors, we generate a suite of color stimuli that are all mutual metamers after bias by the illuminant. The color sensation evoked by these metamers is specified by tristimulus values $\mathbf{T} = \mathbf{A}'\mathbf{W}_\alpha\mathbf{N}_a$. Note that $\mathbf{P}_\alpha\mathbf{N}_b$ is not a true residual or metameric black because two stimuli that differ by $\mathbf{P}_\alpha\mathbf{N}_b$ are metameric only *after* bias by the illuminant. We must bias vector $\mathbf{P}_\alpha\mathbf{N}_b$ by the illuminant to obtain a true residual. Consequently, vector $\mathbf{P}_\alpha\mathbf{N}_b$ may be termed an "antebiased residual."

What is the relationship between matrix **P** and matrix **R?** In the special case of an equal energy illuminant, $\mathbf{W} = \mathbf{I}$, the biased and unbiased color matching functions coincide and the projection matrix **P** is $\mathbf{I} - \mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'$. The difference between any two metamers, **PN**, is a true residual. The fundamental, $\mathbf{N}^* = \mathbf{N} - \mathbf{PN}$, is the part of **N** with no residual, in other words $\mathbf{PN}^* = 0$. To show that $\mathbf{N} - \mathbf{PN}$ has no residual component, we substitute $\mathbf{N} - \mathbf{PN}$ for $\mathbf{N}^*$ in the equation $\mathbf{PN}^* = 0$ and obtain $\mathbf{P}(\mathbf{N} - \mathbf{PN}) = 0$. The projection matrix **P** is idempotent, that is to say, $\mathbf{PP} = \mathbf{P}$. Thus $\mathbf{P}(\mathbf{N} - \mathbf{PN})$ becomes $\mathbf{PN} - \mathbf{PN} = 0$, proving that $\mathbf{N} - \mathbf{PN}$ is the fundamental of **N**. Expanding $\mathbf{N} - \mathbf{PN}$, we have $(\mathbf{I} - \mathbf{P})\mathbf{N}$ or $(\mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}')\mathbf{N}$. Matrix **R** is the projection matrix $(\mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}')$ which strips away the residual and preserves the fundamental: $\mathbf{RN} = \mathbf{N}^*$. Matrix **R** projects into fundamental color space; matrix **P** projects into residual space.

When considering an illuminant other than equal energy,

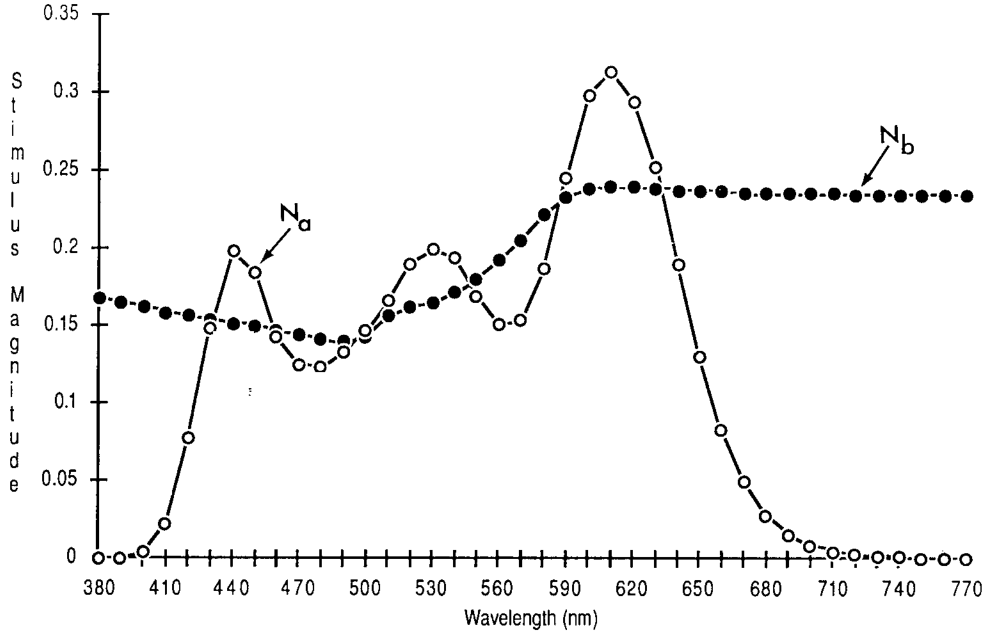

FIG. 1. Two unbiased radiometric functions that are dual metamers with respect to either CIE standard illuminant A or C. Unbiased radiometric function $\mathbf{N}_b$ represents Munsell 5YR 5/2 under an equal energy illuminant. For the respective tristimulus values under each illuminant, see text.

$\mathbf{W}_\alpha$, a biased matrix $\mathbf{R}_\alpha$ can be introduced as $\mathbf{A}_\alpha(\mathbf{A}_\alpha'\mathbf{A}_\alpha)^{-1}\mathbf{A}_\alpha'$. While matrix $\mathbf{P}_\alpha$ produces antebiased residuals, matrix $\mathbf{R}_\alpha$ does not produce antebiased fundamentals, that is, $\mathbf{R}_\alpha\mathbf{N}$ is not a true fundamental after bias by illuminant $\mathbf{W}_\alpha$. Biased matrix $\mathbf{R}_\alpha$ does, however, have a practical use which will be demonstrated subsequently.

The Problem of Multiple Metamers

Prior literature discloses one technique that generates multiple metamers with respect to two illuminants. Takahama and Nayatani[15] employed the variation calculus to isolate a single pair of unbiased color stimuli metameric with respect to CIE illuminants A (tungsten) and D65 (daylight). Another pair with respect to standard illuminants A and C, apparently using the same variation calculus, appears both in Judd and Wyszecki[16] and Wyszecki and Stiles.[17] When these unbiased stimuli are weighted by the first illuminant, the resulting biased color stimuli match, and evoke a certain color sensation; when weighted by the second illuminant, they match again, but evoke a different color sensation. As has been noted by Cohen and Kappauf,[1,2] Takahama and Nayatani contains terms related to matrix-**R** theory, but stops short of conceptualizing the fundamental and residual color spaces.[18] A paper by Worthey investigates a related problem, that is, finding color stimuli that match under one illuminant but differ maximally under another.[19]

How may matrix-**R** operations be extended to multiple metamers with respect to two illuminants, $\mathbf{W}_\alpha$ and $\mathbf{W}_\beta$? Using previously introduced terminology, we seek pairs of unbiased color stimuli, $\mathbf{N}_1$ and $\mathbf{N}_2$, which are metamers after bias by one illuminant, $\mathbf{W}_\alpha$, and which are also metamers after bias by a second illuminant, $\mathbf{W}_\beta$. In the case of the first illuminant, the two metameric biased stimuli are specified by the same triplet of tristimulus values: $\mathbf{T}_\alpha = \mathbf{A}'(\mathbf{W}_\alpha\mathbf{N}_1) = \mathbf{A}'(\mathbf{W}_\alpha\mathbf{N}_2)$. Similarly, for the second illuminant: $\mathbf{T}_\beta = \mathbf{A}'(\mathbf{W}_\beta\mathbf{N}_1) = \mathbf{A}'(\mathbf{W}_\beta\mathbf{N}_2)$ and, in general, $\mathbf{T}_\alpha$ will not equal $\mathbf{T}_\beta$.

Again, we use biased color matching functions to rephrase our problem: $\mathbf{A}_\alpha'\boldsymbol{\Delta}\mathbf{N} = 0$ and $\mathbf{A}_\beta'\boldsymbol{\Delta}\mathbf{N} = 0$. These equations can be combined to form a single matrix equation:

$$\begin{bmatrix} \mathbf{A}_\alpha' \\ \hline \mathbf{A}_\beta' \end{bmatrix} \Delta\mathbf{N} = 0.$$

We define a composite $N \times 6$ matrix, $\mathbf{A}_{\alpha\beta}$, comprising $\mathbf{A}_\alpha$ as the first triplet of columns and $\mathbf{A}_\beta$ as the second triplet of columns. Then the problem acquires the form $\mathbf{A}_{\alpha\beta}'\boldsymbol{\Delta}\mathbf{N} = 0$, nearly identical to the single illuminant case presented earlier. The principal difference is that matrix $\mathbf{A}_{\alpha\beta}$ has rank six instead of three, assuming that all six columns are linearly independent. Consequently, the previously employed mathematical treatment may be applied; projection matrix $\mathbf{P}_{\alpha\beta} = \mathbf{I} - \mathbf{A}_{\alpha\beta}(\mathbf{A}_{\alpha\beta}'\mathbf{A}_{\alpha\beta})^{-1}\mathbf{A}_{\alpha\beta}'$ imposes a relationship between dual metamers $\mathbf{N}_1$ and $\mathbf{N}_2$, such that they differ by $\mathbf{P}_{\alpha\beta}\mathbf{N}$, where $\mathbf{N}$ is an arbitrary set of N numbers. By selecting several arbitrary sets of N numbers, we generate a series of antebiased residuals; added to any given color stimulus, they form a suite of metameric stimuli after bias by either illuminant $\mathbf{W}_\alpha$ or illuminant $\mathbf{W}_\beta$.

We now give geometrical meaning to the entities created. In the case of a single illuminant, $\mathbf{W}_\alpha$, the projection matrix $\mathbf{P}_\alpha$ defines an (N–3)-dimensional subspace containing all antebiased residuals with respect to illuminant $\mathbf{W}_\alpha$. Simi-

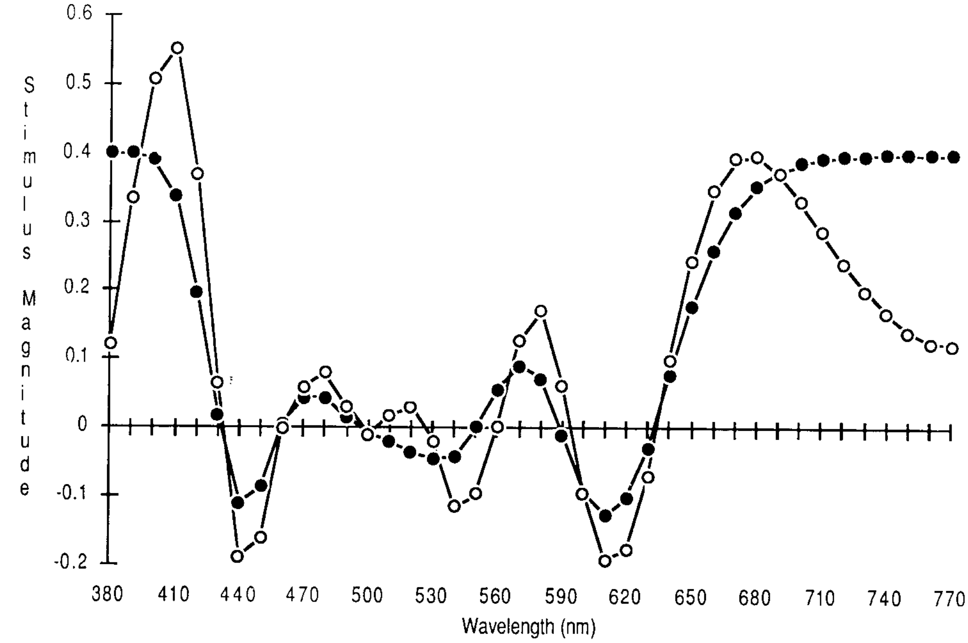

FIG. 2. Two antebiased residuals with respect to CIE illuminants A or C, derived with extended matrix-**R** operations. Multiple metamers, with respect to illuminants A or C, must differ by this type of antebiased residual. (●) Antebiased residual #1; (○) antebiased residual #2.

larly, $\mathbf{P}_\beta$ defines an (N–3)-dimensional subspace containing all antebiased residuals with respect to illuminant $\mathbf{W}_\beta$. Projection matrix $\mathbf{P}_{\alpha\beta}$ defines an (N–6)-dimensional subspace which is the intersection of the two subspaces defined by $\mathbf{P}_\alpha$ and $\mathbf{P}_\beta$.

In addition to matrix $\mathbf{P}_{\alpha\beta}$, we may also form a dually-biased matrix **R**: $\mathbf{R}_{\alpha\beta} = \mathbf{A}_{\alpha\beta}(\mathbf{A}_{\alpha\beta}'\mathbf{A}_{\alpha\beta})^{-1}\ \mathbf{A}_{\alpha\beta}'$. This form of matrix **R** will generate special radiometric functions (but *not* antebiased fundamentals) of the type $\mathbf{N}_a = \mathbf{R}_{\alpha\beta}\mathbf{N}_b$ with the interesting property that $\mathbf{N}_a$ and $\mathbf{N}_b$ differ by the antebiased residual $\mathbf{P}_{\alpha\beta}\mathbf{N}_b$. Consequently, $\mathbf{N}_a$ and $\mathbf{N}_b$ represent two color stimuli that will be metamers when biased by either illuminant $\mathbf{W}_\alpha$ or illuminant $\mathbf{W}_\beta$. This provides a convenient means of generating dual metamers that evoke

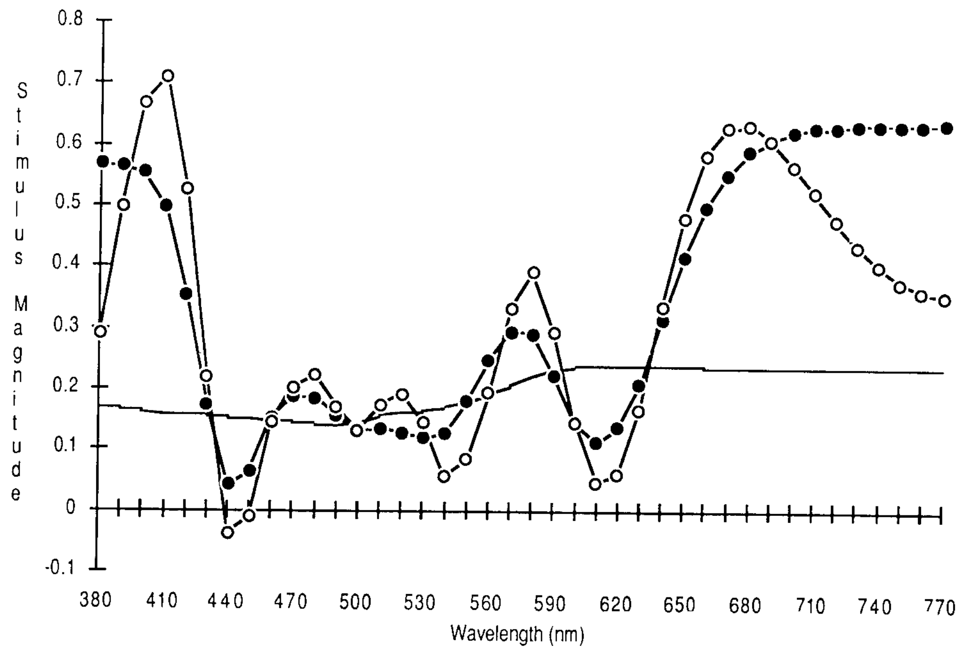

FIG. 3. Three unbiased radiometric functions that are dual metamers with respect to CIE standard illuminants A or C. Radiometric functions #1 (●) and #2 (○) were obtained by adding respectively antebiased residuals #1 and #2 in Fig. 2 to the radiometric function of Munsell 5YR 5/2, shown here as radiometric function #3 (–). All functions of Fig. 3 and Fig. 1, after bias by either illuminant A or C, belong to the same suite of dual metamers

a *specified* color sensation. We select the desired color sensation, say that evoked by a particular Munsell chip, and premultiply its radiometric function by $\mathbf{R}_{\alpha\beta}$. The result is a different radiometric function that evokes the identical sensation as the original Munsell chip when both are biased by either illuminant $\mathbf{W}_\alpha$ or illuminant $\mathbf{W}_\beta$.

We now demonstrate the generation of dual metamers with respect to CIE standard illuminant A and CIE standard illuminant C. We seek a radiometric function that evokes the same sensation as Munsell 5YR 5/2 under both illuminants. Figure 1 illustrates the radiometric function $\mathbf{N}_b$ associated with Munsell 5YR 5/2, and its dual metamer, the function $\mathbf{N}_a = \mathbf{R}_{AC}\mathbf{N}_b$. Matrix $\mathbf{R}_{AC}$ equals $\mathbf{A}_{AC}(\mathbf{A}_{AC}'\mathbf{A}_{AC})^{-1}\mathbf{A}_{AC}'$, where $\mathbf{A}_{AC}$ is the composite $N \times 6$ matrix of biased color mixture functions. These two color stimuli, $\mathbf{N}_a$ and $\mathbf{N}_b$, represent unbiased radiometric functions emitted by two colored objects under equal energy illumination. The objects will match under standard illuminant A [with tristimulus values $(X,Y,Z) = (2.79, 2.30, 0.594)$] or under standard illuminant C [with tristimulus values $(X,Y,Z) = (2.35, 2.22, 2.03)$]. In this paper, all examples refer to the CIE 1964 standard observer.

Premultiplication by $\mathbf{R}_{\alpha\beta}$ provides a pair of physically realizable multiple metamers only if the resulting radiometric function has no negative components, as is the case with single metamers. If negatives appear, we may resort to the more general means of producing multiple metamers, that is, to select members from a suite of antebiased residuals. Since all residuals have negative components, several trials may be necessary to arrive at a physically realizable solution. Figure 2 illustrates two members of a suite of antebiased residuals with respect to CIE standard illuminants A and C. These antebiased residuals are added to $\mathbf{N}_b$ (Munsell 5YR 5/2) yielding two new radiometric functions. These new functions and the original function, shown together in Fig. 3, are mutual metamers after bias by either standard illuminant A or C. These stimuli evoke the identical sensation and are specified by the same tristimulus values as the functions in Fig. 1 after bias. Note that one of the radiometric functions in Fig. 3 has negative components and is not physically realizable.

Multiple Metamers with Respect to More than Two Illuminants

Matrix-**R** operations may be extended to the case of more than two illuminants. The number of biased color matching functions in the composite **A** matrix, and the rank of the corresponding **R** matrix, will increase by three with each addition of another illuminant, while the dimensionality of the biased residual space will simultaneously decrease by three. In principle, the number of independent illuminants is limited by the number of finite width passbands chosen in discretizing the continuous spectrum. This number also determines the number of rows in matrix **A**. Computational accuracy decreases as the biased color matching functions approach linear dependency, giving rise to an ill-conditioned multiply-biased **R** matrix; the determinant of certain matrices will approach zero.

Figure 4 demonstrates a pair of multiple metamers with respect to four different artificial illuminants, illustrated in Fig. 5. The chosen illuminants were sufficiently different to avoid numerical problems associated with near linear dependency. In this example, the number of passbands was increased to 391, to better represent the rapidly fluctuating radiometric function produced. The tristimulus values associated with the metameric pairs under each illuminant are

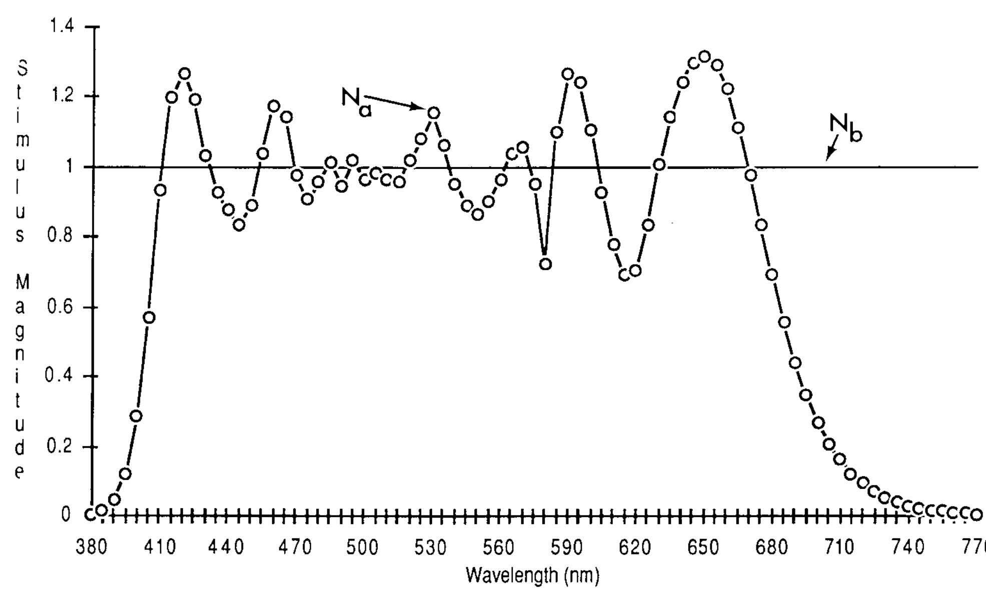

FIG. 4 Two unbiased radiometric functions that are quadruple metamers with respect to four artificial illuminants shown in Fig. 5. Radiometric function $\mathbf{N}_b$ represents an object color stimulus with equal reflectance at all wavelengths. Unbiased radiometric function $\mathbf{N}_a$ was obtained from premultiplication of $\mathbf{N}_b$ by quadruply-biased matrix **R**. The operation is similar to that illustrated in Fig. 1 for two illuminants. For the respective tristimulus values under each illuminant, see text.

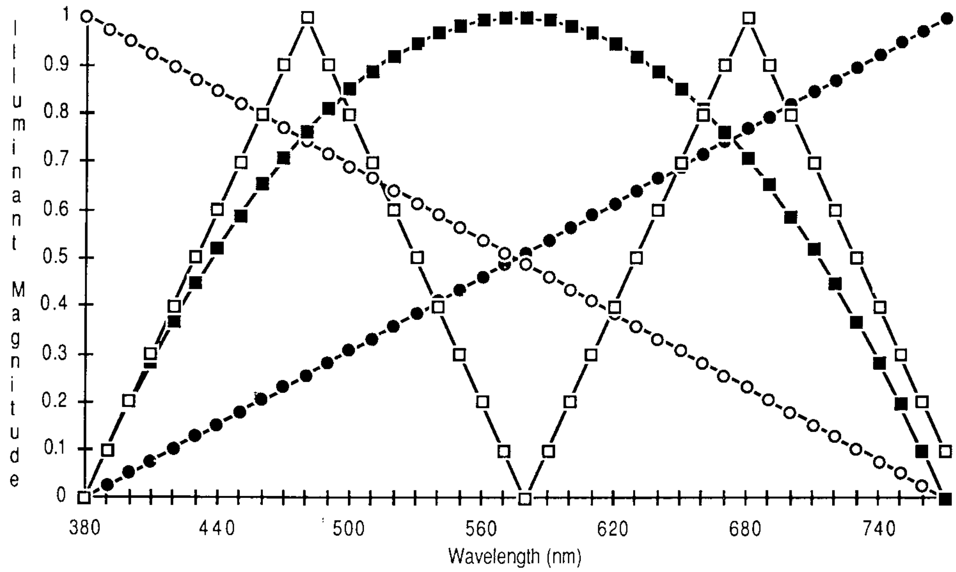

FIG. 5. The unbiased color stimuli of Fig. 4 are multiple metamers after bias by any one of these artificial illuminants. (●) Illuminant #1; (○) illuminant #2; (■) illuminant #3; (□) illuminant #4.

as follows: #1 = (5.62, 5.23, 2.12); #2 = (6.04, 6.44, 9.55); #3 = (10.4, 10.9, 6.77); and #4 = (4.05, 4.58, 7.92).

Summary and Conclusions

We have defined multiple metamers as color stimuli remaining metameric with respect to two or more illuminants. Matrix-**R** operations have been extended to generate multiple metamers for many illuminants, drastically simplifying the required analytical effort.

Strictly speaking, an adequate general computational tool for solving the stated problem is linear programming. It establishes the existence of linear combinations of ante-biased residuals satisfying the above requirement (nonnegative values in the resulting color stimulus). If a solution exists, the weight factors in the obtained linear combination must be appropriately bounded for the result to be physically meaningful.

There exists a far-reaching mutual reciprocity among the four building blocks of color stimuli: object color stimuli, luminous color stimuli, illuminants, and filters. When these blocks are logically permutated in the problem statements and solutions, all formal relations and underlying reasonings and obtained solutions, as presented in the paper, will remain intact. Consider this problem: Given a colored object, find two different illuminants such that the object evokes the same color sensation under either illuminant, that is, the respective two biased stimuli are metamers. This problem is the reciprocal of a prior problem posed and solved in this paper. Similar alternative reformulations for other considered cases are stated easily. For example, the quadruple metamer problem presented in Figs. 4 and 5 may be interpreted as follows: Given the four unbiased color object stimuli shown in Fig. 5, find two illuminants which bias each of the four stimuli such that each pair of resulting biased stimuli (four pairs total) is metameric. Figure 4 is the desired solution.

We may extend matrix-**R** operations to a fifth domain, namely, the observer described by the color matching functions. Observer-specific **A** matrices may be associated with different observers. Metamers may be produced with respect to diverse observers, that is, stimuli that are metameric as sensed by one observer and metameric as sensed by another observer.

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